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Physical properties of materials that determine reversible and irreversible deformations

Yu. A.Alyushin
National University of Science and Technology MISIS, Moscow, 119991, (RUSSIA)
E-mail: alyushin7@gmail.com
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ABSTRACT

The mechanisms of reversible and irreversible deformations are considered using a model of mechanics based on the concepts of space, time and energy. Local energy, as a generalized scalar function, is represented as a linear combination of independent invariants of the equations of motion in the form of Lagrange with the addition of terms that characterize the processes of dissipation and hardening. The use of material density as a General factor of physical properties involved in the mathematical formulation of the concept of "energy", which determines the scale of the energy scale for all types of energy manifested in the movement of solids, is justified. Arguments are given in favor of switching to a new scale of average stresses, including on the basis of comparing changes in potential and elastic energy during deformation under the influence of its own weight. A variant of experimental determination characteristics of the elastic properties is proposed. The possibilities of spontaneous processes of energy transfer from one form to another within one invariant for elastic deformation and with simultaneous change of several invariants for irreversible deformation are noted. An interpretation of the coefficients associated with the physical properties of the material in the field of irreversible deformations is proposed using the unified curve hypothesis and the theory of plastic flow. The determining role of density, heat capacity and coefficient of linear expansion of the material in the processes of deformation and the energy state of particles is proved. © 2020 Knowledge Empowerment Foundation

KEYWORDS

Equations of motion; Lagrange variables; Energy; Physical properties; Reversible and irreversible deformations.

INTRODUCTION

The concept of "physical properties" is ambiguously interpreted in different sources. The reference[1] refers to physical properties of a material that do not depend on the structure and can change without applying external loads, such as density, specific electrical conductivity, coefficient of thermal expansion, magnetic permeability, and lattice parameter. It is separately stipulated that mechanical properties cannot be attributed to physical properties.

According to the mining encyclopedia[2], physical properties include characteristic qualities due to the composition and structure of a substance that are constant under certain external conditions and change naturally with changes in the latter, such as density, hardness, plasticity, etc. In this list, "plasticity" is questionable, since,

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unlike density and hardness, plasticity does not have a quantitative assessment and generally accepted methods of determination.

Metal scientists[3] believe that physical properties characterize the state of the material, as well as its ability to respond to external factors that do not affect the chemical composition of the material. Physical properties include, in addition to magnetic and thermal, flexibility, shrinkage and elongation without specifying the conditions for their determination.

The free encyclopedia[4] refers to them all the properties inherent in a substance outside of chemical interaction, including not only optical, thermal and electrical, but also mechanical.

Analyzing different sources, we can note the uncertainty in relation to the Poisson's ratio, which characterizes the ratio of transverse and longitudinal deformations in linear tension. In works[5,6], it is referred to the physical properties of the material along with the young's modulus, and in textbooks on the resistance of materials[7,8] – to the mechanical characteristics, which include the limits of yield and strength. The peculiarity of the latter is that they can change within 15% or more depending on the features of metal production, storage conditions, testing, etc.

For the sake of certainty in this paper, we will understand the physical properties as scalar stable characteristics of materials, which allow us to calculate directly unobservable energy parameters of the state through the measured kinematic characteristics of motion.

The purpose of this work is to substantiate the physical properties that determine the energy characteristics of particles during the deformation of bodies using a model of mechanics based on the concepts of space, time and energy[9].

BASIC CONCEPTS OF MECHANICS BASED ON THE CONCEPTS OF SPACE, TIME AND ENERGY

The energy model of mechanics[9,10] is based on the statement that the equations of motion

$$x_i = x_i(\alpha_p, t) \quad \textbf{(1)}$$

where $x_i \in (x, y, z)$, $\alpha_p \in (\alpha, \beta, \gamma)$ – Euler and Lagrange variables, respectively, t – time, carry all information about external influences and changes occurring in each particle of the observed mechanical system. Energy as a generalized scalar characteristic of any types of motion[11] must take into account all independent invariant characteristics of the system (1).

The choice of the reference system is subjective, so the invariants can only be defined in terms of derivatives of Euler variables in time and directions. Given the different nature of coordinates $\alpha_p \in (\alpha, \beta, \gamma)$ and time t, two independent operators are used to denote infinitesimal increments of any function $f(\alpha, \beta, \gamma, t)$, as in[9,10]: the operator d for increments in time $\mathrm{d}f(\alpha, \beta, \gamma, t) = f_t \mathrm{d}t$, and the operator δ– for increments in space $\delta f(\alpha, \beta, \gamma, t) = f_\alpha \delta\alpha + f_\beta \delta\beta + f_\gamma \delta\gamma$, where $\delta\alpha, \delta\beta, \delta\gamma$ — are infinitesimal increments of Lagrange variables.

The main information about the state of particles is provided by the Jacobian components

$$\partial x_i / \partial \alpha_p \equiv x_{i,\alpha_p} = \begin{pmatrix} x_\alpha & x_\beta & x_\gamma \\ y_\alpha & y_\beta & y_\gamma \\ z_\alpha & z_\beta & z_\gamma \end{pmatrix} \quad \textbf{(2)}$$

In the most General case, the invariants of system (1) are 3 modules of the vectors displacement u, velocity v, acceleration w, and path s:

$$\xi_1 = |u| = \sqrt{u_x^2 + u_y^2 + u_z^2} = \sqrt{(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2}$$

$$\xi_2 = |v| = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{x_t^2 + y_t^2 + z_t^2} \quad \xi_3 = |w| = \sqrt{w_x^2 + w_y^2 + w_z^2} = \sqrt{x_{tt}^2 + y_{tt}^2 + z_{tt}^2} \quad \xi_4 = s = \int_{t_1}^{t_2} |v| dt \qquad (3)$$

three invariants of the tensor $\partial x_i / \partial \alpha_p \equiv x_{i,\alpha_p}$, that determine the deformation of the particle:

$$\xi_5 = x_\alpha + y_\beta + z_\gamma \qquad \xi_6 = x_\alpha^2 + x_\beta^2 + x_\gamma^2 + y_\alpha^2 + y_\beta^2 + y_\gamma^2 + z_\alpha^2 + z_\beta^2 + z_\gamma^2 \qquad \xi_7 = |x_{i,p}| = R = \delta V / \delta V_0 \qquad (4)$$

three invariants of the strain rate tensor $\partial x_{i,t} / \partial \alpha_p \equiv x_{i,t\alpha_p}$:

$$\xi_8 = x_{t\alpha} + y_{t\beta} + z_{t\gamma} \qquad \xi_9 = x_{t\alpha}^2 + x_{t\beta}^2 + x_{t\gamma}^2 + y_{t\alpha}^2 + y_{t\beta}^2 + y_{t\gamma}^2 + z_{t\alpha}^2 + z_{t\beta}^2 + z_{t\gamma}^2 \qquad \xi_{10} = \left| x_{i,t\alpha_p} \right| \qquad (5)$$

and three time integrals of the three invariants (4) of the strain rate tensor:

$$\xi_{11} = \int \xi_8 \mathrm{d}t \qquad \xi_{12} = \int \xi_9^{1/2} \mathrm{d}t \qquad \xi_{13} = \int \xi_{10}^{1/3} \mathrm{d}t. \qquad (6)$$

The energy carriers are particles, including infinitesimal ones with volume $\delta V_0 = \delta\alpha\delta\beta\delta\gamma$. In the simplest version, the generalized scalar function $\delta E = \delta E(\xi_i)$ can be written as the sum of 13 terms, each of which is represented by the product of the corresponding invariant on the volume δV_0 and a scalar multiplier k_i, that ensures equality of the dimensions of the terms:

$$\delta E = \sum_{i=1}^{13} \delta E_i(\xi_i) = \sum_{i=1}^{13} k_i \xi_i \delta V_0 \qquad (7)$$

Equation (7) assumes the existence of 13 types of energy that characterize in General the movement of absolutely solid or deformable bodies, taking into account various factors manifested in changes in the position, volume and shape of particles.

Since the starting point of various types of energy can be chosen arbitrarily, it is advisable to use the law of conservation of energy as an increment

$$d\delta E = \delta V_0 (d \sum_{i=1}^{13} k_i \xi_i) = 0 \qquad (8)$$

Systems that include interacting bodies that are not affected by bodies from other systems that are not included in the considered system are called isolated (closed). In other words, in an isolated system, the cause of motion may be material objects within the system. Only for isolated systems can the energy conservation law be used in the form (8).

Any part of an isolated system can be considered a separate (closed) subsystem if the action of external causes in relation to it is replaced by mathematical images equivalent in their influence on the equations of motion, called external forces. An isolated system differs from a dedicated one by the absence of external forces.

The particles inside anybody cannot represent an isolated system, so the law of conservation of energy for a particle must be supplemented with a function that takes into account the interactions at its boundaries. Denoting the energy equivalent of external influences by $d\delta E_e$, instead of (8) for the particle should be written

$$d\delta E = \delta V_0 (d \sum_{i=1}^{13} k_i \xi_i) - d\delta E_e = 0 \qquad (9)$$

Equation (9) is equivalent to the first beginning of thermodynamics for a continuous medium particle[5], according to which the work of external forces is spent on changing the kinetic, potential, elastic or other types of energy of the particle. All known forms of the laws of motion[12] should be considered as special cases of equation (9).

The scalar coefficients k_i included in equations (7) – (9) must characterize the physical properties of the material or medium in which the movement occurs. Since they, together with the invariants ξ_i, are included in the law of conservation of energy (8), only 12 can be independent.

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The peculiarity of assumption (7) with a linear combination of invariants is reduced to the mandatory appearance of a common coefficient multiplier k_i (i = 1, 2 ..., 13), which can be considered as a basic energy property, since it determines the scale of the energy scale in all its manifestations.

In contrast to classical mechanics, in the energy model there is no fundamental difference between absolutely solid and deformable bodies. In the absence of deformation, part of the terms in equation (9) turns to 0 due to constant invariants.To determine the basic energy property, it is sufficient to consider the free fall of a solid body in the gravitational field of the Earth with a change in two types of energy[9,10]. If you do not take into account the air resistance, the system can be considered isolated, the movement occurs under the law of conservation of energy in the form (8).

By orienting the z-axis to the center of the Earth, we write equations (1) as

$$x = \alpha,\ \ y = \beta,\ \ z = \gamma + u_z(\gamma, t)$$

The increments of potential $\mathrm{d}\delta E_1$ and kinetic $\mathrm{d}\delta E_2$ energy will be

$$\mathrm{d}\delta E_1 = -k_1 \delta V_0 z_t \mathrm{d}t < 0$$

$$\mathrm{d}\delta E_2 = k_2 \delta V_0 \mathrm{d}(v_z^2) = 2k_2 \delta V_0 z_{tt} z_t \mathrm{d}t > 0$$

Law of conservation of energy (8)

$$\mathrm{d}\delta E_1 + \mathrm{d}\delta E_2 = -k_1 z_t \delta V_0 \mathrm{d}t + 2k_2 z_t z_{tt} \delta V_0 \mathrm{d}t = 0 \qquad \mathbf{(10)}$$

for the considered version of the motion determines the relationship between the coefficients $k_1 = 2k_2 z_{tt}$. Using the generally accepted notation for acceleration of free fall $z_{tt} = g$, we get $k_1 = 2k_2 g$. In equation (10), the coefficients k_1 and k_2 characterize the properties of the particle and should be assumed

$$k_1 = \rho_0 g\ , \qquad k_2 = \rho_0 / 2\ , \qquad \mathbf{(11)}$$

then for a body with volume V_0 and mass m we get the generally accepted expressions for kinetic and potential energy

$$E_\mathrm{k} = mv^2/2,\ \ \Delta E_\mathrm{p} = mg\Delta z.$$

Thus, the basic energy property for potential (in the gravitational field of the Earth) and kinetic energy is the density of the material P0. There is reason to believe that it should be included as a multiplier in all other types of energy through the coefficients k_i (i = 3, 4, ..., 13). This confirms the use of body mass in estimating the energy cost of friction (k_4) in the movement of absolutely solid bodies in classical mechanics

$$d\delta E_4 = k_4 \delta V_0 d\xi_4 = \rho_0 g f_{fr} \delta V_0\, ds \qquad \mathbf{(12)}$$

where $k_4 = \rho_0 g f_{fr}$, f_{fr} – is the coefficient of friction.

The base multiplier of the physical propertiesshould be taken into account when choosing experimental and other methods for determining physical properties that lead to equation (9)

The experience of solving various problems with absolutely solid and deformable bodies[10], including with vibrations[13,14], allows us to reduce the number of invariants that affect the energy state of particles and moving bodies in General on the basis of sufficiently strong arguments.

First of all, this concerns accelerations. It is usually noted[5] that the equations of motion (1) are doubly differentiable functions. This does not exclude the mention of the invariant ξ_3 in the concept of energy (7). However, accelerations are discontinuous functions by their nature, the time derivative of accelerations has no physical meaning, and therefore they should be excluded from equations (8) and (9).

Indeed, in the gravitational field of the Earth, a body resting at a certain height does not have acceleration, but if it begins to fall, the acceleration immediately acquires the value g = 9.81 m/s^2. In this case, the energy of the body does not change, this moment corresponds to the appearance of speed and, accordingly, the beginning of changes in potential and kinetic energy. It is possible to determine accelerations from equation (9), since they participate in the conservation law as time derivatives of the velocities included in the invariant ξ_2.

The linear invariant ξ_5, which determines the average relative length of the projections of the three base faces of an infinitesimal parallelepiped on the coordinate axes that coincide with them in the direction in the initial state, should also be excluded from further consideration. The sum of elements of the main diagonal of the Jacobian (2) is indeed invariant with respect to the rotation of the coordinate axes[15], but it changes when the particle is rotated as a rigid whole.

In particular, when the body rotates relative to the z axis with the equations of motion

$$x = \alpha\cos(\omega t) - \beta\sin(\omega t) \qquad y = \alpha\sin(\omega t) + \beta\cos(\omega t) \qquad z = \gamma$$

and Jacobian (2)

$$x_{i,\alpha_p} = \begin{pmatrix} \cos(\omega t) & -\sin(\omega t) & 0 \\ \sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

the invariant ξ_5, in contrast to ξ_6 and ξ_7, depends on the angle of rotation of the particle as a rigid whole,

$$\xi_5 = 2\cos(\omega t) + 1 \qquad \xi_6 = 3 \qquad \xi_7 = 1$$

which should not affect the strain energy. As a consequence, the invariants ξ_8 and ξ_{11} must also be excluded from equations (7) – (9) in the future.

External forces can be of any nature: gravitational, magnetic, or mechanical. In particular, for an infinitesimal particle inside the body, the energy equivalent of external influences δE_e on the movements of its boundaries can be determined by the scalar product of the acting forces $\delta\mathbf{P}$ and displacements $\mathrm{d}\mathbf{r}$ [9]

$$\mathrm{d}\delta E_e = \sum(\delta\mathbf{P}\cdot\mathrm{d}\mathbf{r}) = \sum(\delta\mathbf{P}\cdot\mathbf{v})\mathrm{d}t \qquad \textbf{(13)}$$

Taking into account possible changes in forces and velocities on opposite faces, assuming all functions are differentiable and set in Lagrange variables, we obtain[9,10] from equation (13) for the rate of change in the specific energy of external influencesω_e, taking into account the rule of summation by a repeating index

$$\omega_e = \mathrm{d}\delta E_e/(\delta V_0\mathrm{d}t) = \tau_{pi}x_{i,t\alpha_p} + x_{i,t}\partial\tau_{pi}/\partial\alpha_p \qquad \textbf{(14)}$$

The Lagrange stresses τ_{pi} form an unsymmetric tensor of the second rank. Representing the increment of the energy of external influences through the volume density of its rate of change (14), the law of energy conservation (9) takes the form

$$\mathrm{d}\delta E = \delta V_0(k_1\xi_{1,t} + k_2\xi_{2,t} + k_4\xi_{4,t} + k_6\xi_{6,t} + k_7\xi_{7,t} + k_9\xi_{9,t} + k_{10}\xi_{10,t} + k_{12}\xi_{12,t} + k_{13}\xi_{13,t} - \omega_e)\mathrm{d}t = 0. \qquad \textbf{(15)}$$

it can be considered as a form of energy balance.

The properties determined by the coefficients k_6 and k_7 should determine the energy changes of the particle in the area of elastic deformations. Taking into account the previously made remarks about the exclusion invariant ξ_5 and the relations given in[9,10], the coefficientk_7 is an additive component of the average voltage

$$\sigma = 2k_6\xi_6/(3R) + k_7 \qquad \textbf{(16)}$$

it can only affect the selection of the starting point of the average voltage scale.

In[9,17], there are reasons to consider average stresses as a measure of the volume energy density and not to accept them as equal to 0 in the initial state of the particle. In particular, the feasibility of switching to a new average stress scale is consistent with the Boyle-Marriott law for ideal gases, which coincides with the law of elastic volume change $\sigma = 3K\varepsilon$, if we consider the volume elasticity modulus K as the actual pressure p_0 in the material, and the hydrostatic stress σ as an increment of the internal pressure Δp

$$p_0V_0 = p_1V_1 = (p_0 + \Delta p)(V_0 + \Delta V) \qquad \text{or} \qquad \Delta p = \sigma = -p_0\Delta V/V_0 = 3K\varepsilon$$

An argument in favor of transferring the beginning of the average stress scale can be a comparison of changes in elastic and potential energy when the rod is stretched under its own weight[10].

Consider two pivotally connected rods *OA* and *AB* of length *L*, located in the initial state in the horizontal plane $x = 0$ (the x axis is directed to the center of the Earth) with the coordinates of the axes of the hinges *O(0, 0), A(0, L), B(0, 2L)*. At some point, the hinge *B* begins to move to the origin. When the points *O* and *B* coincide, the rods *OA* and *AB* take a vertical position. The deformation of rods under the action of their own weight is described by the well-known[7,8] relations

$$\sigma_{xx} = \rho g x \qquad \varepsilon_{xx} = \rho g x / E$$

where E is Young's modulus. For the total length L the strain energy is

$$\Delta E_{def} = 0,5\int_V \sigma_{xx}\varepsilon_{xx}\delta V = 0,5 mgL\frac{\rho g L}{3E} \tag{17}$$

The change in the potential energy of the position due to gravitational forces during the transition from an undeformed horizontal state to a deformed vertical state is

$$\Delta E_{pot} = 0,5 mgL \tag{18}$$

The difference between the right parts of equations (17) and (18) determines the multiplier (ρg/3E), which for steel (ρ = 7.8 g/cm^3, g = 9.8 m/s^2, E = 200 GPa) has the order 10^{-7} 1/m.

There is no discrepancy when comparing the energy change in the new scale. To fulfill the condition of energy invariance with respect to the choice of the velocity reference system[9,10], the equation must be fulfilled

$$x_{i,t}\left(\frac{\partial \tau_{pi}}{\partial \alpha_p} + \rho_0 g_i - \rho_0 x_{i,tt}\right) = 0 \tag{19}$$

For a new average stress scale with a single modulus of elasticity

$$\tau_{pi} = 2k_6 x_{i,p} \tag{20}$$

it is converted to the form

$$x_{i,t}\left(\frac{\partial x_{i,p}}{\partial \alpha_p} + \frac{\rho_0}{2k_6} g_i - \frac{\rho_0}{2k_6} x_{i,tt}\right) = 0 \tag{21}$$

Taking the hypothesis of plane sections, according to which $x_\beta = x_\gamma = 0$, for the equation $x = x(\alpha_p, t)$ when $\tau_{\alpha x} = 2k_6 x_\alpha$ we get

$$\partial^2 x / \partial\alpha^2 = -\rho_0 g / (2k_6) = -\psi \qquad \text{or} \qquad x = -0,5\psi\alpha^2 + C_1\alpha + C_2$$

As boundary conditions for determining the integration constants C_1 and C_2, we use the assumption that there are no displacements at the upper end ($x = 0$ at $\alpha = 0$), and deformations at the lower end ($x_\alpha = 1$ at $\alpha = L_0$). Then for the equations of motion (1) under tension from its own weight we get

$$x = \alpha[1 + \psi(L_0 - \alpha/2)] \qquad y = \beta[1 - \mu\psi(L_0 - \alpha)] \qquad z = \gamma[1 - \mu\psi(L_0 - \alpha)]$$

where μ is the Poisson coefficient, with Jacobian (2)

$$x_{i,\alpha} = \begin{pmatrix} x_\alpha & x_\beta & x_\gamma \\ y_\alpha & y_\beta & y_\gamma \\ z_\alpha & z_\beta & z_\gamma \end{pmatrix} = \begin{pmatrix} 1+\psi(L-\alpha) & 0 & 0 \\ \beta\mu\psi & 1-\mu\psi(L-\alpha) & 0 \\ \gamma\mu\psi & 0 & 1-\mu\psi(L-\alpha) \end{pmatrix}$$

and the volume density of elastic energy by changing the invariant ξ_6

$$\delta E_{def} / \delta V_0 = k_6\{[1+\psi(L_0-\alpha)]^2 + 2[1-\mu\psi(L_0-\alpha)]^2 + 2\mu^2\psi^2(\beta^2+\gamma^2)\}$$

As noted above, the ratio $\psi = \rho_0 g/(2k_6)$ is on the order of 10^{-7} 1/m. Ignoring the squares ψ^2, we get

$$\delta E_{def} / \delta V_0 = 2k_6 \psi (L_0 - \alpha)(1-2\mu) = \rho_0 g (L_0 - \alpha)(1-2\mu)$$

or, after integrating by volume with mass m,

$$E_{def} = \frac{1}{2} mgL_0 (1-2\mu) \quad \textbf{(22)}$$

The resulting ratio cannot be considered accurate. In particular, the equations of motion do not satisfy the boundary conditions $y_\beta = 1$ and $z_\gamma = 1$ on the side surfaces of the deformable body. However, the change in the potential energy of the position of deformable rods (modulus)

$$\Delta E_{p\alpha} = \rho_0 g F_0 \int_0^{L_0} x \delta\alpha = \rho_0 g F_0 \int_0^{L_0} \alpha[1 + \psi(L_0 - \alpha/2)]\delta\alpha = 0{,}5 mgL_0 (1 + 2\psi L_0/3) \quad \textbf{(23)}$$

it has the same order as the elastic energy change. Both results (22) and (23) coincide in the absence of transverse deformations ($\mu = 0$).

As an additional argument in favor of switching to a new scale of average stresses (20), we can compare the dependence of the Cauchy average stress from the energy model of mechanics, given in[9],

$$3\sigma R = \tau_{pi} x_{i,p} = 2k_6 \xi_6 \quad \textbf{(24)}$$

with experimental data for hydrostatic compression when

$$\sigma = 2k_6 R^{-1/3}$$

After switching to the generally accepted scale by shifting the starting point by $2k_6$ we get

$$\sigma = 2k_6 (R^{-1/3} - 1) = 2k_6 (1 - R^{-1/3})/R^{1/3} \quad \text{or} \quad R = 1/[\sigma/(2k_6) + 1]^3 \quad \textbf{(25)}$$

The ratio (25) at $2k_6 = 3K$ and K=169 GPa differs from that established experimentally for iron in the range up to 300 GPa[18]

$$R = 1 + 5{,}286 * 10^{-6} \sigma + 0{,}8 * 10^{-10} \sigma^2$$

with an error of no more than 0.016%.

For linear stretching with equations of motion

$$x = \alpha(1 + \varepsilon_{xx}) \qquad y = \beta(1 - \mu\varepsilon_{xx}) \qquad z = \gamma(1 - \mu\varepsilon_{xx}) \quad \textbf{(26)}$$

from equality (24) for the generally accepted scale (after shifting the average stress scale by the value $2k_6$), we get $\sigma_{xx} = 2k_6 \varepsilon_{xx}(1-2\mu)$ or $\sigma_{xx} = E\varepsilon_{xx}$, if we accept $2k_6 = 3K$ and take into account the dependence between the elasticity modules $E = 3K(1-2\mu)$.

Taking into account the above arguments, $k_7 = 0$ is accepted in the following statement. As follows from equation (16), the new scale provides for an mean stress $\sigma_0 = 2k_6$ in the initial state ($\xi_6 = 3, R = 1$). The multiplier k_7 should be excluded from further analysis as not related to the physical properties that determine the energy state of the particles. But this conclusion cannot be considered a sufficient reason for excluding coefficients k_{10}, k_{13} with invariants ξ_{10} and ξ_{13}. Instead of equation (15) for the energy balance, we get

$$d\delta E = \delta V_0 (k_1 \xi_{1,t} + k_2 \xi_{2,t} + k_4 \xi_{4,t} + k_6 \xi_{6,t} + k_9 \xi_{9,t} + k_{10} \xi_{10,t} + k_{12} \xi_{12,t} + k_{13} \xi_{13,t} - \omega_e) dt = 0 \quad \textbf{(27)}$$

The number of physical properties required to describe the energy state of particles has been reduced from 13 to 8.

From the considered examples, it follows that the coefficient k_6 has the order of the volume elasticity modulus, but, unfortunately, they do not give its exact value and do not allow us to distinguish the base factor (density ρ_0) in it.

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POSSIBLE METHODS FOR EXPERIMENTAL DETERMINATION OF THE k_6 MODULE

Methods for experimental determination of physical properties included in equation (27) should meet the conditions of uniform deformation and high accuracy of determining energy costs. It is the inhomogeneity of the deformed state that did not allow us to find the relationship between k_6 and the tensile density on its own weight.

It is preferable to consider the method of deformation in which the linear and cubic invariants of the tensor (2) preserve the initial values $k_5=3$ and $k_7=1$, then the energy of the particles changes only due to the quadratic invariant ξ_6.

This option can be implemented by compressing a spring[10] with the radii of the wire r_w and the Central layer of turns R_s in the global coordinate system of the observer $x_i \in (x, y, z)$, whose z axis coincides with the axis of the spring. Mutually orthogonal axes x and $y$are located in a horizontal plane, orthogonal to the z axis, the x axis passes through the center of the section of the lower turn with coordinates (R_s, 0,0).

The initial coordinates of any particle can be expressed through the helix angle ψ_0 from the x-axis to the considered cross section, the radius of the particle r_0 in this cross section and the angle ζ_0 characterizing the position of the particle relative to a horizontal plane

$$x_0 = \alpha = (R_s + r_0 \cos\zeta_0)\cos\psi_0 \qquad y_0 = \beta = (R_s + r_0 \cos\zeta_0)\sin\psi_0 \qquad z_0 = \gamma = h_0\psi_0/(2\pi) + r_0 \sin\zeta_0 \quad \textbf{(28)}$$

where h_0 is the step of the helix in the initial state. Independent arguments in the system (28) can vary within $0 \le r_0 \le r_w$, $0 \le \psi_0 \le 2\pi N$ and $0 \le \zeta_0 \le 2\pi$, N is the number of turns. The radius of the Central layer of the turns R_s is the same for all particles and can only change as the spring $R_s(h)$ is compressed.

In an arbitrary loaded state, the position of any particle can be written in the same way, but taking into account the current values of the angle ψ, the radius of the particle in the cross section r, the angle ζ and radius of the Central fibers of the spring turns R_{si}

$$x = (R_{si} + r\cos\zeta)\cos\psi \qquad y = (R_{si} + r\cos\zeta)\sin\psi \qquad z = h\psi/(2\pi) + r\sin\zeta \quad \textbf{(29)}$$

where h is the step of the spring turns in the current state. It is possible to change $0 \le r \le r_{wi}$ if the wire diameter changes during the deformation process, but for other arguments the range is assumed to be unchanged $0 \le \psi \le 2\pi N$, $0 \le \zeta \le 2\pi$.

To determine the derivatives included in the Jacobian of type (2), we use the General rules for differentiating implicit functions, considering the parameters of the auxiliary system as functions of Lagrangian coordinates, for example, for the function $x(\alpha, \beta, \gamma, t)$

$$x(\alpha,\beta,\gamma,t) = \{R_s(t) + r(\alpha,\beta,\gamma,t)\cos[\zeta(\alpha,\beta,\gamma,t)]\}\cos[\psi(\alpha,\beta,\gamma,t)]$$

then

$$\partial x/\partial\alpha \equiv x_\alpha = r_\alpha \cos\zeta\cos\psi - r\zeta_\alpha \sin\zeta\cos\psi - \psi_\alpha (R_s + r\cos\zeta)\sin\psi \quad \textbf{(30)}$$

where the lower indices of the parameters r, ψ, ζ correspond to the partial derivatives of the specified Lagrange variables.

The derivatives included in the right part can be found by considering the spatial increments of Lagrange variables in the system (28) by changing the parameters of the auxiliary system δr_0, $\delta\psi_0$ and $\delta\zeta_0$

$$\delta\alpha = \cos\psi_0 \cos\zeta_0 \delta r_0 - \sin\psi_0 (R_s + r_0 \cos\zeta_0)\delta\psi_0 - r_0 \cos\psi_0 \sin\zeta_0 \delta\zeta_0$$

$$\delta\beta = \sin\psi_0 \cos\zeta_0 \delta r_0 + \cos\psi_0 (R_s + r_0 \cos\zeta_0)\delta\psi_0 - r_0 \sin\psi_0 \sin\zeta_0 \delta\zeta_0$$

$$\delta\gamma = \sin\zeta_0 \delta r_0 + h_0/(2\pi)\delta\psi_0 + r_0 \cos\zeta_0 \delta\zeta_0$$

By solving this system of linear equations with respect to increments δr_0, $\delta\psi_0$, and $\delta\zeta_0$, we obtain inverse dependencies of the auxiliary system's increments on the Lagrange variables ' increments. They determine the derivatives required for equations of type (30) that coincide with the multipliers at increments of the corresponding Lagrange variables

$$\delta r_0 = (\cos\psi_0 \cos\zeta_0 + \eta_0 \sin\psi_0 \sin\zeta_0)\delta\alpha + (\sin\psi_0 \cos\zeta_0 - \eta_0 \cos\psi_0 \sin\zeta_0)\delta\beta + \sin\zeta_0 \delta\gamma$$

$$\delta r_0 = (\cos\psi_0 \cos\zeta_0 + \eta_0 \sin\psi_0 \sin\zeta_0)\delta\alpha + (\sin\psi_0 \cos\zeta_0 - \eta_0 \cos\psi_0 \sin\zeta_0)\delta\beta + \sin\zeta_0 \delta\gamma$$

$$\delta\psi_0 = -(\sin\psi_0 \delta\alpha - \cos\psi_0 \delta\beta)/\chi_0$$

$$\delta\zeta_0 = (-\cos\psi_0 \sin\zeta_0 + \eta_0 \sin\psi_0 \cos\zeta_0)\frac{\delta\alpha}{r_0} - (\sin\psi_0 \sin\zeta_0 + \eta_0 \cos\psi_0 \cos\zeta_0)\frac{\delta\beta}{r_0} + cos\zeta_0 \frac{\delta\gamma}{r_0}$$

where $\chi_0 = R_s + r_0 \cos\zeta_0$, $\eta_0 = h_0/(2\pi\chi_0)$. Using the common notation for partial derivatives, we get

$$\partial r_0/\partial\alpha = (\cos\psi_0 \cos\zeta_0 + \eta_0 \sin\psi_0 \sin\zeta_0) \qquad \partial r_0/\partial\beta = \sin\psi_0 \cos\zeta_0 - \eta_0 \cos\psi_0 \sin\zeta_0$$

$$\partial r_0/\partial\gamma = \sin\zeta_0 \qquad \partial\psi_0/\partial\alpha = -\sin\psi_0/\chi_0$$

$$\partial\psi_0/\partial\beta = \cos\psi_0/\chi_0 \qquad \partial\zeta_0/\partial\alpha = (-\cos\psi_0 \sin\zeta_0 + \eta_0 \sin\psi_0 \cos\zeta_0)/r_0$$

$$\partial\zeta_0/\partial\beta = -(\sin\psi_0 \sin\zeta_0 + \eta_0 \cos\psi_0 \cos\zeta_0)/r_0 \qquad \partial\zeta_0/\partial\gamma = \cos\zeta_0/r_0$$

For derivatives that are included in equations of type (30), it is necessary to take into account possible changes in the parameters r, ψ, ζ when the spring is deformed, using the dependencies

$$R_{si} = R_{si}(h, R_s) \qquad r = r(h, r_0) \qquad \psi = \psi(h, \psi_0) \qquad \zeta = \zeta(h, \zeta_0)$$

Then the derivatives, in comparison with those given in equation (30), will have a new multiplier of the type $\partial f/\partial f_0$, for example, for the ones used in the definition below $\partial x/\partial\alpha \equiv x_\alpha$

$$\partial r(r_0, h)/\partial\alpha = \partial r/\partial r_0 * \partial r_0/\partial\alpha = \frac{\partial r(r_0, h)}{\partial r_0}(\cos\psi_0 \cos\zeta_0 + \eta_0 \sin\psi_0 \sin\zeta_0)$$

$$\partial\zeta/\partial\alpha = \partial\zeta/\partial\zeta_0 * \partial\zeta_0/\partial\alpha = \frac{\partial\zeta(\zeta_0, h)}{\partial\zeta_0}\frac{1}{r_0}(-\cos\psi_0 \sin\zeta_0 + \eta_0 \sin\psi_0 \cos\zeta_0)$$

$$\partial\psi/\partial\alpha = \partial\psi/\partial\psi_0 * \partial\psi_0/\partial\alpha = \partial\psi/\partial\psi_0 \frac{-\sin\psi_0}{\chi_0}$$

This results in the bulkiness of all subsequent equations, for example, instead of (30) after replacement r_α, ψ_α, and ζ_α,

$$x_\alpha = \{\partial r/\partial r_0\}(\cos\psi_0 \cos\zeta_0 + \eta_0 \sin\psi_0 \sin\zeta_0)\cos\zeta \cos\psi - \{\partial\psi/\partial\psi_0\}\frac{-\sin\psi_0}{\chi_0}(R_{si} + r\cos\zeta)\sin\psi -$$
$$-\{\partial\zeta/\partial\zeta_0\}(-\cos\psi_0 \sin\zeta_0 + \eta_0 \sin\psi_0 \cos\zeta_0) r/r_0 \sin\zeta \cos\psi$$

If we now assume that the derivatives in curly brackets and the current coordinates are close to their initial values

$$\partial r/\partial r_0 \approx 1,\ \partial\psi/\partial\psi_0 \approx 1,\ \partial\zeta/\partial\zeta_0 \approx 1,\ r \approx r_0,\ \psi \approx \psi_0,\ \zeta \approx \zeta_0, \qquad \textbf{(31)}$$

then the right side of the previous equation is converted to a constant

$$x_\alpha = \eta_0 \sin\zeta \cos\zeta \sin\psi \cos\psi + \cos^2\psi(\sin^2\zeta + \cos^2\zeta) - \eta_0 \sin\zeta \cos\zeta \sin\psi \cos\psi + \sin^2\psi = \cos^2\psi + \sin^2\psi = 1$$

In the same way, ignoring the size change in the results, except for the spring height, we get the final dependencies for the elements of the Jacobian (2), for $R_s = R_{si} = const$

$$\frac{\partial x}{\partial \alpha} = 1,\ \frac{\partial y}{\partial \beta} = 1,\ \frac{\partial z}{\partial \alpha} = \frac{h - h_0}{2\pi\chi}\sin\psi,\ \frac{\partial z}{\partial \beta} = -\frac{h - h_0}{2\pi\chi}\cos\psi,\ \frac{\partial z}{\partial \gamma} = 1$$

Compression of helical cylindrical springs is accompanied by shear deformations with equations of motion

$$x = \alpha \qquad y = \beta \qquad z = \gamma + \frac{h - h_0}{2\pi(R_s + r\cos\zeta)}(\alpha\sin\psi - \beta\cos\psi) \qquad \textbf{(32)}$$

Note that if we find the equations of motion based on systems (28), (29) and assumptions (31), instead of (32) we get the equations of motion of an absolutely rigid body without any deformations

$$x = \alpha \qquad y = \beta \qquad z = \gamma + \psi\frac{h - h_0}{2\pi}$$

Equations (32) satisfy the condition of independence of energy from the choice of the velocity reference system (21), which for the considered case (in the absence of gravitational and inertial forces) leads to Laplace equations for each of the coordinates[16,17]

$$x_{\alpha\alpha} + x_{\beta\beta} + x_{\gamma\gamma} = 0 \qquad y_{\alpha\alpha} + y_{\beta\beta} + y_{\gamma\gamma} = 0 \qquad z_{\alpha\alpha} + z_{\beta\beta} + z_{\gamma\gamma} = 0$$

The volume of particles remains unchanged $R = | x_{i,\alpha_p} | = \delta V / \delta V_0 = 1$, and the volume density of the acquired elastic energy is determined by a change in the invariant ξ_6

$$\Delta\xi_6 = \left(\frac{h - h_0}{2\pi\chi}\right)^2 = \left(\frac{h - h_0}{2\pi(R_s + r_0\cos\zeta_0)}\right)^2$$

If $R_s >> r_w$, the deformation can be considered the same across the entire wire section

$$\delta\xi_6 = \left(\frac{h - h_0}{2\pi\chi}\right)^2 = \left(\frac{h - h_0}{2\pi R_s}\right)^2$$

The volume-integral elastic energy is (N is the number of turns)

$$E_{def} = k_6\frac{(h - h_0)^2}{2R_s}r_w^2 N$$

The work of external forces spent on spring deformation can be determined by the relative compression of the spring, the change in the potential energy of the loading body, or the height of the rebound of a body with a known mass previously located on the end surface of the compressed spring.

For experimental studies, a helical plate spring with a rectangular cross-section (with a height greater than the width) is preferred, for which the calculation of the deformed state does not differ from the above, only the area of determining variables in both coordinate systems used changes. The accuracy of the result is improved by reducing the volume with maximum errors compared to helical cylindrical springs with a round cross-section of the turns.

PHYSICAL PROPERTIES IN THE FIELD OF IRREVERSIBLE DEFORMATIONS

To specify the meaning of the coefficients in the right part of equation (7), which characterize irreversible deformations, consider the terms with invariants (5) – (6), which are included in the energy balance (27). In the absence of a rigid rotation of particles, the invariant ξ_9 practically coincides in meaning and value with the square of the intensity of the strain rates H[19]

$$\xi_9 = H^2 = x_{t\alpha}^2 + x_{t\beta}^2 + x_{t\gamma}^2 + y_{t\alpha}^2 + y_{t\beta}^2 + y_{t\gamma}^2 + z_{t\alpha}^2 + z_{t\beta}^2 + z_{t\gamma}^2 \quad (33)$$

To assess the role of this invariant in the energy balance (27) , we can use the theory of plastic flow in the classical version of deformable solid mechanics, according to which the stress and strain rate deviators are proportional[19,20]

$$\frac{\tau_{pi} - \delta_{pi}\sigma_0}{x_{i,tp} - \delta_{pi} s_0} = \frac{\tau_e}{H} = \lambda \quad (34)$$

where τ_e is the intensity of the tangential stresses determined via the quadratic invariant of the deviator[19], s_0 is the equivalent of σ_0 for the tensor $x_{i,tp}$, $\delta_{pi} = 1$ for $\tau_{\alpha x}$, $\tau_{\beta y}$, $\tau_{\gamma z}$ and $x_{t\alpha}$, $y_{t\beta}$, $z_{t\gamma}$, for all other cases, $\delta_{pi} = 0$. The right part (33) coincides with the square of the intensity of the strain rate H at $s_0 = 0$.

The scalar coefficient of proportionality λ with dimension [Pa*s] can be determined experimentally for processes with uniform deformation by the consumed specific power[20]

$$\omega = T_d H = \lambda H^2 \quad (35)$$

(35) at a known strain rate H. Taking into account the relations (34), the energy component in equation (7) with the invariant ξ_9 can be written as

$$\delta E_9 = k_9 \xi_9 \delta V_0 = k_9 H^2 \delta V_0 = \omega(k_9 / \lambda)\delta V_0 \quad (36)$$

The expression in brackets (k_9/λ) characterizes not only the material properties, but also the deformation conditions. On this basis, it can be transferred to the category of mechanical properties.

It is important that the invariant ξ_9 with dimension [1/s^2] does not characterize a state, but a process. It differs from 0 only in cases when the surrounding particles are in motion. If the speed of the surrounding particles is equal to 0 or the same as the motion of rigid bodies are derived from the velocities $x_{i,t\sigma}$ and invariant ξ_9 turn to 0. The coefficient k_9 is the dimension [Pa*s^2], the characteristics of the materials with this dimension in modern mechanics of deformable bodies no. These features give reason to exclude it from the balance (27), but they can play an important role in dynamic processes and for accounting for viscous properties, including in the case of reversible deformation.

Difficulties in interpreting the summand with the invariant ξ_9 lead to a logical replacement of mathematical images of some invariants, without distorting their meaning and the basic concept of energy (7). An additional argument for the possibility of such a replacement can be the previously used transition from the velocity modulus (3) to its square for kinetic energy. Now the situation is reversed, there are reasons to accept the quadratic invariant of the deviator instead of (33)

$$\xi_9 = H = [(x_{t\alpha} - y_{t\beta})^2 + (y_{t\beta} - z_{t\varphi})^2 + (z_{t\gamma} - x_{t\alpha})^2 + x_{t\beta}^2 + x_{t\gamma}^2 + y_{t\alpha}^2 + y_{t\gamma}^2 + z_{t\alpha}^2 + z_{t\beta}^2]^{1/2} \quad (37)$$

The above considerations, including the possibility of using the theory of plastic flow (34) and the calculation of the deformation power (35), remain valid, but the final expression for the corresponding local energy invariant takes the form instead of (36)

$$\delta E_9 = k_9 \xi_9 \delta V_0 = k_9 H \delta V_0 = (k_9 / \lambda) T_d \delta V_0 \quad (38)$$

The dimension of k_9 for the modified invariant (37) coincides with the dimension of the proportionality coefficient of the theory of plastic flow λ [Pa*s], which can be determined experimentally[21].

The logic of the proposed transition from (33) to (37) becomes clearer if we additionally replace the expression of invariant $\xi_{10} = \left| x_{i,t\alpha_p} \right|$ given in block (4) with a close-meaning derivative of the invariant $\xi_7 = R$

$$\xi_{10} = \xi_{7,t} = R_t = (\delta V / \delta V_0)_t \qquad \delta E_{10} = k_{10} \xi_{10} \delta V_0 = k_{10} R_t \delta V_0 \quad (39)$$

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In this case, the terms with the invariants ξ_9 and ξ_{10} do not characterize States, but a process, and it is possible to combine them in the energy balance (27), as mutually complementary in the transitions of the energy of shape change (invariant ξ_9) to the energy of volume change (invariant ξ_{10}) mentioned in classical mechanics of a deformable solid.

The quadratic invariant ξ_6 can be represented as a sum of summands[9,10]

$$\xi_6 = x_\alpha^2 + x_\beta^2 + x_\gamma^2 + y_\alpha^2 + ...z_\gamma^2 = e_\alpha^2 + e_\beta^2 + e_\gamma^2 = \Gamma_e^2 = 3e^2 + \Gamma^2 \quad \textbf{(40)}$$

In the right part, the squares of the ratio of the edge lengths before δl_0 and after δl are used, initially oriented in the direction of the corresponding axes

$$e_p^2 = (\delta l / \delta l_0)_p^2 = x_p^2 + y_p^2 + z_p^2, \qquad p \in (\alpha, \beta, \gamma)$$

average value of the ratio of edge lengths e_p,

$$e = (e_\alpha + e_\beta + e_\gamma)/3$$

and the standard deviation of the relative edge lengths e_p from the average value e

$$\Gamma^2 = (e_\alpha - e)^2 + (e_\beta - e)^2 + (e_\gamma - e)^2$$

The structure of the formula (40) allows the transition of part of the elastic energy from one form $3e^2$ to another Γ^2 without changing the invariant ξ_6. Transitions are not restricted by any other conditions and are reversible. They are realized in reality at free fluctuations[13,14] without the participation of external energy sources.

The possibility of energy transition associated with the invariants ξ_9 and ξ_{10} has a different nature. The parts of energy that have passed from one type to another are reflected at each moment of time or after the completion of the process in equations of type (7) and (8) in terms with other invariants (ξ_6, ξ_{12}, ξ_{13}). The peculiarity of the process is that ξ_9 is always positive, and ξ_{10} can have any sign. If the sum of local energies (38) and (39) is equal to 0, the process can proceed spontaneously, but only with a decrease in the volume of particles (for $H>0$, $R_t>0$, the sum will not be equal to 0).

In particular, for linear stretching with equations of motion (26) at values (up to $\varepsilon_{x,t}$)

$$H \approx \varepsilon_{x,t}(1+2\mu^2)^{1/2} \qquad R_t \approx (1-2\mu)\varepsilon_{x,t}$$

from the condition $k_9 H + k_{10} R_t = 0$ we get

$$k_9 / k_{10} = -R_t / H = -(1-2\mu)/(1+2\mu^2)^{1/2} = (2\mu - 1)/(1+2\mu^2)^{1/2}$$

Since the scalar properties are positive, the Poisson coefficient μ must be greater than 0.5. This is consistent with the example given in[9] of irreversible deformation under linear tension, when the second phase of deformation with the volume returning to the original value is possible only at $\mu > 0.5$.

The invariant $\xi 12$ essentially coincides with the concept of accumulated strain used in classical mechanics (the Odquist hardening parameter)[19]

$$\xi_{12} = \int_0^t H dt = \Lambda \quad \textbf{(41)}$$

where the intensity of the strain rate H determines the equation (37). The parameter Λ characterizes the hardening of the material and the degree of use of its plastic properties (in the initial state $\Lambda = 0$). Separately, we note that for integration in the right part (41), the equations of motion (1) must be written in the form of Lagrange, since the accumulated deformation should not be determined for a fixed point in the observer's space (Euler), but for a fixed particle of the medium.

Difference between accumulated deformation (41) and effective

$$\Gamma_e = \sqrt{\Gamma_e^2} = (x_\alpha^2 + x_\beta^2 + x_\gamma^2 + y_\alpha^2 + y_\beta^2 + y_\gamma^2 + z_\alpha^2 + z_\beta^2 + z_\gamma^2 - 3)^{1/2} \quad \textbf{(42)}$$

it consists in the fact that the Γ_e can increase and decrease over time, while the accumulated strain Λ, by analogy with the path ξ_4, always increases, regardless of the type of loading trajectory.

Processes for which the Odquist parameter Λ coincides with the effective strain (42) are called monotonic[21]. Typically, such deformations occur under the action of a single external force, such as linear tension, simple shear, or torsion, when all deformations increase in proportion to a single parameter. In this case, the loading trajectory in the deformation space has the form of a straight line connecting the initial and final States (movement along the shortest path). In all other cases, $\Lambda > \Gamma_e$ and the more complex the deformation path, the greater the difference between Λand Γ_e.

Like the invariant ξ_4, an additional operator Δ must be used to increment the energy of the particle corresponding to the invariant ξ_{12}

$$\Delta\delta E_{12} = k_{12}\xi_{12}\delta V_0 = k_{12}\delta V_0\int H dt = k_{12}\Lambda\delta V_0 \quad \textbf{(43)}$$

The operator Δ in the right part of the equation is not necessary, since by definition the accumulated deformation Λ in the initial state is equal to 0.

The unified curve hypothesis of classical mechanics[19] assumes the replacement of a constant coefficient *G* (modulus of shift) in the condition of proportionality of stress and strain deviators for an elastic region with a positive function $g(\Lambda)$, called the modulus of plasticity[21]. An analog of the unified curve hypothesis in the space of Lagrange variables is the relations

$$\frac{\tau_{pi} - \delta_{pi}\sigma_0}{x_{i,\alpha_p} - \delta_{pi}e_0} = \frac{\tau_{\alpha y}}{y_\alpha} = \frac{T}{\Gamma} = g(\Lambda)$$

According to the unified curve hypothesis[19], the product $T = g(\Lambda)\Gamma$ determines the stress intensity *T*, which coincides with the quadratic invariant of the stress deviator. Then the local energy associated with the k_{12} property in the region of irreversible plastic deformations $\delta E_{12}/\delta V_0$ corresponds to an increment of the volume energy density due to the accumulated deformation Λ in comparison with the initial state. Equation (43) takes the form

$$\Delta\delta E_{12}/\delta V_0 = k_{12}\xi_{12} = k_{12}\Lambda = g(\Lambda)\Lambda$$

For models of ideal plasticity, linear and power-law isotropic hardening, the expressions are used instead of the coefficient k_{12}, respectively

$$k_{12} = g(\Lambda) = \sigma_T = const \qquad k_{12} = g(\Lambda) = C\Lambda \qquad k_{12} = g(\Lambda) = C\Lambda^n \quad \textbf{(44)}$$

For the elastic region of deformation $\Lambda = \Gamma$, we obtain Hooke's law for the shear $T = G\Gamma$ (*G* is the shear modulus, Γ is the shear strain intensity). In the plastic region, the constant *C* should be determined based on experimental studies, usually by the curves of hardening under linear tension beyond the elastic limit[21].

Replacing the invariant ξ_{10} with R_t changes the invariant ξ_{13}

$$\xi_{13} = \int_0^t R_t \mathrm{d}t = R - 1 = \delta V/\delta V_0 - 1 = \Delta\delta V/\delta V_0 = 3\varepsilon$$

which coincides with the relative change in the volume of the particle 3ε in magnitude and sign. Thus, the term δE_{13} in equation (7) will take into account instead of the invariant ξ_7 the increment of energy that characterizes the current value of the particle volume in comparison with the initial state

$$\delta E_{13} = k_{13}\xi_{13}\delta V_0 = 3\varepsilon k_{13}\delta V_0$$

If we take the coefficient k_{13} equal to the modulus of bulk elasticity of the material *K*, then $\delta E_{13}/\delta V_0$ we determine the average Cauchy stress, which in the energy model of mechanics corresponds to the increment of the average stress in the current state compared to the initial one $\sigma_0 = 2k_6$, which follows from equation (16) for

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the new scale of average stresses. This is the meaning given to the average Cauchy stress by the analogy of the laws of elastic volume change and Boyle-Marriott used above. It is also noted that from the analogy of laws, as well as in meaning $\delta E_{13}/\delta V_0$, the volume elasticity modulus K should be considered as the actual pressure acting in the material. In addition, we note that $\delta E_{13}/\delta V_0$ it acquires the sign ε, increases with increasing volume, and decreases otherwise. This effect, which does not agree with the usual representation of classical mechanics, was noted in[13] various signs of changes in the volume of body particles during longitudinal free vibrations did not require additional energy from external influences, if the total volume of the body remains unchanged if the total volume of the body.This confirms the possibility of classifying the volume elasticity modulus as a physical property

$$k_{13} = K \quad (45)$$

but, unfortunately, without a base multiplier.

TEMPERATURE EFFECTS IN DEFORMATION PROCESSES

As noted above, energy is invariant with respect to the type of external influences. Gravitational and mechanical are considered above, and we will focus on the possible temperature effects in the deformation processes.

Dissipation, accompanied by thermal effects with the loss of part of the energy through the heat generated, leads to the irreversibility of the processes and, therefore, can only occur in a plastic state. But the temperature increase due to external sources can occur in both elastic and plastic areas.

To take into account the thermal effects, it is necessary to Supplement the energy balance (27) with a new term related to external influences, in the form of heat flow ω_T

$$d\delta E = \delta V_0 (k_1\xi_{1,t} + k_2\xi_{2,t} + k_4\xi_{4,t} + k_6\xi_{6,t} + k_9\xi_{9,t} + k_{10}\xi_{10,t} + k_{12}\xi_{12,t} + k_{13}\xi_{13,t} - \omega_e - \omega_T)dt = 0 \quad (46)$$

which for the considered particle can be represented as

$$\omega_T = \mathrm{d}\delta E_T / (\mathrm{d}t\delta V_0) = c_0\rho_0 dT/dt = c_0\rho_0 T_t$$

where c_0, ρ_0, T are the heat capacity, density, and temperature, respectively.

First, consider the case when the particle is in its original undeformed state. For an isotropic material, the thermal energy coming from external sources leads to a homogeneous expansion with the equations of motion[9,18]

$$x_i = \alpha_i(1+\alpha_T\Delta T) \qquad x_\alpha = 1+\alpha_T\Delta T \quad (47)$$

where $\alpha_Ò$ is the coefficient of linear expansion, with an increment of the particle's energy

$$d\delta E_T/\delta V_0 = c_0\rho_0 dT \quad (48)$$

For homogeneous deformation, the invariants of the equations (47) included in the energy balance (46) determine the equations

$$\xi_6 = 3(1+\alpha_T\Delta T)^2 \qquad \xi_9 = H = \sqrt{3}\alpha_T T_t \qquad \xi_{10} = R_t = [(1+\alpha_T dT)^3]_t = 3\alpha_T T_t$$

$$\xi_{12} = \int H dt = \Lambda = \sqrt{3}\alpha_T\,|\Delta T| \qquad \xi_{13} = R - 1 = (1+\alpha_T\Delta T)^3 - 1 \approx 3\alpha_T\Delta T$$

Change in volume energy density

$$d\delta E/\delta V_0 = k_6 d\xi_6 + k_9 d\xi_9 + k_{10} d\xi_{10} + k_{12} d\xi_{12} + k_{13} d\xi_{13}$$

taking into account the increment of invariants up to dT is

$$d\delta E/\delta V_0 = [sign(dT)(6k_6 + 3k_{13}) + \sqrt{3}k_{12}]\alpha_T\,|dT| + (\sqrt{3}k_9 + 3k_{10})\alpha_T dT_t \quad (49)$$

Earlier, when discussing the local energy (38) and (39) with the invariants ξ_9 and ξ_{10}, it was noted that the energy of changing the shape of δE_9 can be converted into the energy of changing the volume of δE_{10}, with subsequent adjustment of other types of energy included in the energy balance. With uniform heating, both invariants ξ_9 and ξ_{10} increase and the mentioned transition is impossible. The sum of the terms $d\delta E_9$ and $d\delta E_{10}$ will be 0 if only the second time derivative of temperature is 0. Excluding heat shock for the conditions under consideration, which allows $T_{tt} \neq 0$, for the energy balance (46) taking into account $\sigma_0 = 2k_6$, (45) and (48) we obtain

$$sign(dT)(3\sigma_0 + 3K - c_0\rho_0 / \alpha_T) + \sqrt{3}k_{12} = 0$$

Since the first term changes sign during heating and cooling, and the coefficient k12 cannot be negative, the equality holds if only in the initial state $k_{12} = 0$. This corresponds to uniform heating, when there is no hardening, for any law of hardening (44) $k_{12} = 0$ follows. The condition of the energy balance remains equal

$$\sigma_0 + K = c_0\rho_0 / (3\alpha_T) \qquad \textbf{(50)}$$

The terms in the left part were interpreted above as measures of the volume energy density of particles in the initial state. It can be argued that both parts of this equality must be constants of materials. This is confirmed by the Grueneisen law established based on experiments[21]. It is especially important that this equality, as well as the properties (11), includes the base multiplier – the density of the material ρ_0.

It is noted above that the properties of k_9 and k_{10} can contribute to energy transitions with the condition being met $k_9 H + k_{10} R_t = 0$. For each process, the kinematic parameters involved in this equality can take independent values: the intensity of the strain rate H characterizes the shape change, and the time derivative of the Jacobian R_t – the volume change. It is interesting to note that under hydrostatic loading, when $H = 0$ and $R_t \neq 0$, the transition of the energy of volume change to the energy of shape change is excluded, the condition $k_9 H + k_{10} R_t = 0$ is not fulfilled. When thermal effects occur the mentioned equality must be supplemented with a summand

$$k_9 H + k_{10} R_t + c\rho_0 \Delta T = 0 \qquad \textbf{(51)}$$

which determines the dissipated part of the energy and removes restrictions on the ratio of properties k_9 and k_{10}. Equation (51) assumes that a similar term $d\delta E_T = c\rho_0 T_t \delta V_0 dt$ should be included in equation (46), since temperature becomes a new parameter of the energy state of particles, which is not provided for in the equations of motion (1).

Taking the intensity of the shear strain rate (37) as the invariant ξ_9, and the dissipative function (34) as the coefficient k_9, the energy condition for the transition of the strain energy to temperature takes the form

$$\tau_e + k_{10} R_t + c\rho_0 \Delta T = 0$$

It can be argued that the energy $\delta E_{10} = k_{10} R_t \delta V_0$ released by a particle when its volume changes can be spent either on increasing the stress intensity to a value $\tau_e = | k_{10} R_t |$ without changing the temperature of the particle ($\Delta T = 0$), if $\tau_e < \tau_s$, or on increasing the temperature ($\Delta T > 0$), if the stress intensity has reached the limit value $\tau_e = \tau_s$.

In General, when thermal effects may occur during deformation due to external or internal sources, the energy balance should be considered in the form (46) with an additional term $d\delta E_T = c\rho_0 T_t \delta V_0 dt$. Included in the right part, the invariants must take into account the actual deformed state of the particle.

DISCUSSION AND CONCLUSIONS

The energy model does not contradict classical mechanics and brings clarity to the understanding of the basic laws associated with motion. As the experience of solving various problems shows[13-18], the transition to a new scale of average stresses and one modulus of elasticity reduces mathematical difficulties and allows us to obtain new results, including on the features of the energy state of particles and the body as a whole.

The above mentioned physical property ratios $2k_6=3K$, $\sigma_0 = 2k_6$. Equation (50) allows us to obtain concrete results using the density of the material, which, as noted in section. 2, is mandatory for all scalar coefficients

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included in the right part of equation (7). For the volume elasticity modulus *K*, the calculated data can be compared with the table data[18,23], if we go to the energy scale of specific heat in equation (50) and take $\sigma_0 = 0$[9], then
$K = c_0\rho_0/(3\alpha_T)$

The results (TABLE 1) have the same order, the ratio depends on the type of material. The reason for deviations may be the frequently used calculation of the volume elasticity modulus *K* through young's modules *E* or the shear *G* using the Poisson's ratio (for example, for lead in TABLE 1[18]). Additional properties may play a significant role. The greatest differences were observed for metals with aFCC lattice.

TABLE 1

Metal	Gridtype	Density ρ_0	Heat capacity c_0	Linear exp.coef α_T	$k_6 = c_0\rho_0/(2\alpha_T)$	*Kcalc* $c_0\rho_0/(3\alpha_T)$	*Ktabl*[18]	*Ktabl/Kcalc*
		kg/m^3	N*м/kg*C	1/^{o}C*10^6	GPa	GPa	GPa	1
Aluminum	FCC	2700	870	23,9	49,14	32,76	75,8	2,31
Copper	FCC	8960	383	16,5	103,99	69,33	137,6	1,98
Titanium	HCC	4500	520	8,5	137,65	91,76	107	1,16
Nickel	FCC	8900	443	13,3	148,22	98,81	161	1,63
Lead	FCC	11300	128	29,3	24,68	16,45	46	2,80
Zinc	HCC	7100	385	15	91,12	60,74	60	0,99
Iron	BCC	7900	483	11,7	163,06	108,71	169	1,55

FCC – face-centered cubic lattice, BCC – body-centered cubic lattice, HCC – hexagonal compact crystal lattice

For the coefficient k_{12} in classical mechanics, there are 3 versions of the formulas (44), taking into account the features of hardening[19,20]. In the new mean stress reference system, $2k_6$ can be adopted instead of *C*, which allows the base factor to be included in this physical property, for example, for power hardening

$$k_{12} = 2k_6\Lambda^n = (c_0\rho_0/\alpha_T)\Lambda^n$$

where Λ should be determined by the methods known in classical mechanics for calculating the accumulated strain. In this case, k_{12} is not a constant, but there is no such requirement in equation (7). The coefficient of friction in the property k_4 is also not a constant. It is possible that the coefficient k_{12} will require an adjustment that takes into account the dependence of the heat capacity on temperature and deformation.

The need to take into account the sign of the terms in equation (49) allows us to assert that the successive alternation of heating and cooling, as well as stretching and compression, cannot be considered reversible processes. They can lead to changes in the properties of materials identified with coefficients k_{12}, k_{13}. An example is the Bauschinger effect[19,20], which is manifested in increasing the yield strength of a material due to pre-deformation of the opposite sign.

It was noted above that the properties of k_9 and k_{10} can contribute to energy transitions with the condition being met $k_9H + k_{10}R_t = 0$. For each process, the kinematic parameters involved in this equality can take independent values: the intensity of the strain rate *H* characterizes the shape change, and the time derivative of the Jacobian R_t – the volume change. It is interesting to note that under hydrostatic loading, when $\acute{I} = 0$ and $R_t \neq 0$, the transition of the energy of volume change to the energy of shape change is excluded, the condition is not fulfilled. When thermal effects occur the mentioned equality must be supplemented with a summand

$$k_9H + k_{10}R_t + c\rho_0\Delta T = 0 \qquad (51)$$

which determines the dissipated part of the energy and removes restrictions on the ratio of properties k_9 and k_{10}.

Equation (51) suggests that a similar term $d\delta E_T = c\rho_0 T_t \delta V_0 dt$ should be included in equation (46), since temperature becomes a new parameter of the energy state of particles, which is not provided for in the equations of motion (1). Additional research is needed to make final decisions on these properties.

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As a result, in equation (7), the energy state of a particle is determined only by 3 physical properties of the material: the density ρ_0, the heat capacity c_0, and the linear expansion coefficient α_T (or volumetric compression coefficient $\beta_T = 3\alpha_T$) of the material. Moreover, the basic density is that which determines the kinetic energy of the particle ($k_2 = \rho_0 / 2$). In other coefficients, the density should be considered as the main factor, and all the others – as additional characteristics of the considered type of local energy. In particular, $k_1 = \rho_0 g$ it characterizes the features of the medium in which the movement occurs (gravitational, magnetic, electric fields or their absence, as in the case of spring deformation). The coefficient $k_4 = \rho_0 g f_{fr}$ assumes the energy consumption at the contact of two bodies, the coefficient of friction f_{fr} can take into account the ratio of the properties of the materials of these bodies. When absolutely solid bodies are moving, it is possible to change 4 types of energy, characterized by coefficients k_1, k_2, k_4 and losses on heating (dissipation) by analogy with equation (51).

For a model with a single modulus of elasticity, if we take k_7=0, the energy state of the particles is determined by coefficients k_6, k_{13}, which differ from the average stress in the initial state of the particle $\sigma_0 = c_0\rho_0 / \alpha_T$ only by numerical coefficients

$$k_6 = \sigma_0 / 2 = c_0\rho_0 / (2\alpha_T), \qquad k_{13} = K = \sigma_0 / 3 = c_0\rho_0 / (3\alpha_T)$$

The coefficients k_9 and k_{10} are not specified, they do not affect the energy state of the particles, and they only determine energy transitions associated with changes in shape, volume, and dissipation according to equation (51).

The use of a single elastic modulus k_6 in the field of reversible deformations allows ambiguous combinations of mean deformations e and standard deviation Γ at a fixed value of elastic deformation Γ_e^2 (40). The system can accept various configurations without additional external influences, including those with extreme values of e and Γ. the Possibility of spontaneous transition of the energy of shape change to the energy of volume change (with or without dissipation) is justified when analyzing the components of the local energy of particles with invariants ξ_9 and ξ_{10}. This allows us to propose a mechanism for the transition to irreversible deformations in the form of a cyclic process with the accumulation of elastic energy in the first phase, usually with an increase in the volume of the particle, and the dissipation of excess energy in the second phase of the cycle with the return of the particle volume to its original value[9].

The valve that starts the process of converting a reversible deformation into an irreversible one is located in the term δE_{13}. According to equation (51), dissipation ($T_t>0$) is only possible when the volume of $R_t<0$ decreases, since $H>0$. The resulting strain rate intensity H is integrated into the accumulated strain Λ with the result displayed in the summand δE_{12}.

The key to actuating the valve can be an increase in the standard deviation Γ, allowed in the elastic state by the invariant (40). As follows from equation (52), for positive by definition e and Γ, an increase in the standard deviation $\Gamma_t>0$ leads to a negative value $e_t< 0$ and, accordingly, a decrease in the volume $R_t< 0$.

The stress level σ_k according to equation (16) and the value R_k at which the valve is triggered determine the new value of the average stress $\sigma_0 + \sigma_k = \sigma_{0k}$ instead of the previous σ_0, which determines the new level of volumetric energy density and hardening with the possible manifestation of the Bauschinger effect.

According to the existing terminology, the k_{12} property should be referred to as mechanical, as it contains, in addition to physical properties, an additional multiplier, and characterizes the resistance to external loads. Explicitly, the Poisson coefficient, Young's modules E and shift G are not included in the physical properties according to the energy model of mechanics.

In total, 8 coefficients of the equation (7) contain only 3 physical properties (TABLE 2).

TABLE 2

k_1	k_2	k_4	k_6	k_7	k_{12}	k_{13}
$\rho_0 g$	$\rho_0/2$	$\rho_0 f_{fr}$	$c_0\rho_0 / (2\alpha_T)$	0	$(c_0\rho_0 / \alpha_T)\, \Lambda^n$	$(c_0\rho_0 / (3\alpha_T)$

Taking into account the initial values of the invariants (all but k_6 are equal to 0), in the initial state, the volume and mass energy densities are

$$\delta E_0 / \delta V_0 = 3k_6 = 1{,}5\sigma_0 = 3c_0\rho_0 / (2\alpha_T) \qquad \delta E_0 / \delta m = 1{,}5c_0 / \alpha_T$$

The specification of physical properties included as scalar coefficients in the main energy equations does not affect the General relations described in[9,18]. The equations of motion in both elastic and plastic regions must satisfy the General differential equation (19), which is transformed for a model with a single elastic modulus k_6 to the form (21). They are used in the study of energy features of free vibrations[13] and resonance[14].

CONCLUSIONS

Only 3 physical properties (density, heat capacity, and linear expansion coefficient) determine the behavior of materials (elastic modulus) and the energy state of particles under reversible deformations. In the field of irreversible deformations, the dependence between the quadratic invariants of stress tensors and strain rates according to the theory of plastic flow of classical mechanics of a deformable solid plays a determining role. The results obtained can be used in selecting criteria for the development of new materials and technologies for their processing. These criteria should take into account the dependence of density, heat capacity, and volume compression on temperature and strain.

REFERENCES

[1] Metals and alloys. Guide. Under the editorship of Y. P. Solntsev, NPO “Professional”, NGO “Peace and family”, St. Petersburg, **(2003)**.
[2] Mountain encyclopedia in 5 volumes Ed., Soviet encyclopedia, **(1987)**.
[3] G.N.Teplukhin, A.V.Kropanov; The Metallography and heat treatment: Proc. manual / St. Petersburg State Technical University, Saint Petersburg, 169 **(2011)**.
[4] *ru.wikipedia.org*
[5] L.I.Sedov; A Course in Continuum Mechanics, Groningen, Wolters-Noorhoff, **(1971)**.
[6] Yu.N.Rabotnov; Mechanics of a deformable solid, Fizmatgiz, Moscow, 744 **(1979)**.
[7] V.I.Feodosiev; Resistance of materials: Textbook. for higher education institutions, Bauman Moscow state technical University, Moscow, 592 **(1999)**.
[8] N.M.Belyaev; Resistance of materials, Nauka, Moscow, 856 **(1965)**.
[9] Y.A.Alyushin; A New concept in mechanics based on the concepts of space, time and energy, Phys. Mesomech, **21(3)**, 59-69 **(2018)**.
[10] Yu.A.Alushin; Energy Foundations of Mechanics: A Handbook for Students, Mashinostroenie, Moscow, 192 **(1999)**.
[11] Aristotle; Essays. Physics, Mir, Moscow, **4**, **(1975)**.
[12] R.Feynman, R.Leighton, M.Sands; Feynman lectures on physics. Mir, Moscow, 219 **(1965)**.
[13] Y.A.Alyushin; Energy characteristics of free vibration in elastic bodies, Phys.Mesomech, **22(3)**, 77-87 **(2019)**.
[14] Yu.A.Alushin; Energy basis of resonance in elastic bodies, Phys.Mezomekh, **22(5)**, 42-53 **(2019)**.
[15] Yu.A.Alushin; Mechanics of deformation processes in the space of Lagrange variables, Mashinostroenie, Moscow, 136 **(1997)**.
[16] Yu.A.Alushin; Solid Mechanics in Lagrangian coordinates: A Handbook for Students, Mashinostroenie, Moscow, 192 **(2012)**.
[17] Yu.A.Alushin; Energy bases of mechanics. Lambert Academic Publishing, 281 **(2016)**.
[18] G.W.C.Kaye, T.H.Laby; Tables of Physical and Chemical Constants and some Mathematical Functions. Longman, London, **(1973)**.
[19] L.M.Kachanov; Fundamentals of plasticity theory, Nauka, Moscow, 420 **(1969)**.
[20] V.L.Kolmogorov; Stresses, Deformations, Fracture, Metallurgy, Moscow, 460 **(1970)**.
[21] L.D.Landau, E.M.Livshits; Statistical Physics, Nauka, Moscow, **1**, 456 **(1973)**.